Standard Model in adS slice with UV-localized Higgs field

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Abstract

We discuss five-dimensional Standard Model in a slice of adS space-time with the Higgs field residing near or on the UV brane. Allowing fermion fields to propagate in the bulk, we obtain the hierarchy of their masses and quark mixings without introducing large or small Yukawa couplings. However, the interaction of fermions with the Higgs and gauge boson KK excitations gives rise to FCNC with no built-in suppression mechanism. This strongly constrains the scale of KK masses. We also discuss neutrino mass generation via KK excitations of the Higgs field. We find that this mechanism is subdominant in the scenarios of spontaneous symmetry breaking we consider.

1 Introduction and summary

Five-dimensional theories with the Standard Model fields living in adS slice attract considerable interest, especially due to their possible connection to adS/CFT correspondence [1, 2, 3]. Originally, theories of this sort focused on the problem of the hierarchy between the Planck and electroweak scales [4]. Later on, it has been understood that they can explain the hierarchy of fermion masses and quark mixings without introducing large or small parameters into the original action [5, 6, 7, 8, 9].

AdS slice is a solution to the Einstein equations in 5-dimensional (5D) space-time with two gravitating branes. Upon fine-tuning the brane tension and 5D cosmological constant, one obtains the metric

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 \ , \tag{1}$$

where k is the adS curvature, y denotes the coordinate of the fifth warped dimension, which is S^1/Z_2 orbifold of size R. Two branes are placed at $y = 0$ and $y = \pi R$. These are ultraviolet (UV, $y = 0$) and infrared (IR, $y = \pi R$) branes, respectively. We choose the 4-dimensional (4D) Minkowski metric as $\eta_{\mu\nu} = (1, -1, -1, -1)$. This setup is known as the Randall-Sundum model of type one (RS1) [4]. In the original RS1 model, only gravity is supposed to propagate in the 5D space-time, while the SM fields reside on the IR brane. However, this is not the only possibility, as one can allow all particles or some of them propagate in the bulk.

In that case, the 5D fields can be expanded in the tower of the Kaluza–Klein (KK) modes, and the zero modes are associated with the SM fields. The hierarchy of fermion masses and quark mixings is then obtained by an appropriate choice of the fermion localization in the bulk.

It is most common to assume that the Higgs field is localized on or near the IR brane, for reviews see, e.g., Refs. [10, 11]. In that case, the zero modes of light fermions are localized near the UV brane, while zero modes of heavy quarks are localized towards the IR brane. The smallness of the SM Yukawa couplings is then due to the small overlaps of the zero modes of the light fermions and the Higgs field. Masses of KK modes of the gauge and Higgs fields (if the latter lives in the bulk) are constrained by the requirement of FCNC suppression. In models with the Higgs field localized on or near the IR brane, this constraint is fairly weak: excited KK modes can have masses of order 10 TeV. This is due to the so called RS–GIM mechanism [12, 13], which is built-in: since the zero modes of light fermions are localized near the UV brane, their overlaps with the KK modes of the gauge and Higgs fields are exponentially small. Moreover, introducing additional symmetries, it is possible to relax the constraints on KK masses down to about 3 TeV [14, 15, 16, 17]. Thus, effects of new physics can be observed in experiments at LHC.

According to the holographic picture [18, 19, 20], every bulk zero mode field corresponds to an eigenstate in the dual 4D theory, which is a mixture of elementary source and composite CFT fields. If the bulk zero mode is localized towards the UV brane, the massless eigenstate of the dual theory is predominantly the source field. Conversely, the dual interpretation of a bulk zero mode localized towards the IR brane is a state that is predominantly a CFT bound state. If the Higgs field is confined to the IR brane, it is interpreted as a pure CFT bound state in the dual theory. The top-quark is mostly a CFT bound state, while light fermions are mostly elementary.

In this paper we turn this picture upside down and consider a scenario with the bulk Higgs field localized near or on the UV brane. Without introducing the hierarchy in the parameters of the original action, we show that the realistic pattern of fermion masses and quark mixings can be obtained in this case as well. However, the overall picture is quite different. Namely, light quarks and right leptons are localized near the IR brane to have small overlaps with the UV-localized Higgs field and, consequently, small Yukawa couplings. Hence, many light SM fields are mostly CFT composites in the dual picture, while heavy fields are predominantly elementary.

The IR localization of light fermions introduces, however, the FCNC problem. Indeed, the KK excitations of the Higgs field and bulk gauge bosons also live near the IR brane. Thus, their wave functions have large overlaps with the wave functions of light fermions. From the point of view of the effective 4D theory, this means that the corresponding couplings are of order one. Then the only parameter one can use for suppressing FCNC is the mass scale of KK excitations. The latter must be very high to satisfy the existing constraints coming from kaon mixing. We will see that the constraint on the KK scale is particularly strong for the Higgs field living in the bulk.

A way to avoid FCNC mediated by the Higgs KK excitations is to localize the Higgs field

on the UV brane. Then there are no Higgs KK excitations at all. In that case the dominant source of FCNC is the exchange by the KK excitations of the gauge fields. Although the constraints here are less severe, the allowed scale of the KK excitations is still beyond the experimental reach.

We also discuss neutrino masses of the Dirac type. It is straightforward to obtain them via the interaction with the zero mode of the Higgs field. An alternative possibility, which would probably be more interesting, would be the neutrino mass generation via the interaction with the KK excitations of the Higgs field. If it worked, the smallness of the neutrino masses would be due to the suppression of the vacuum expectation values of the heavy KK Higgs modes, rather than due to small effective 4D Yukawa couplings. In the particular scenarios we consider, this mechanism is subdominant, however: the neutrino interactions with the Higgs zero mode are always strong enough to generate the main contribution to the neutrino masses.

This paper is organized as follows. In Section 2 we discuss possible scenarios of spontaneous symmetry breaking in the 5D Standard Model with the Higgs field localized towards the UV brane. In Section 3 we show that realistic 4D fermion masses and quark mixings can be obtained without introducing small or large parameters in the 5D action. There we also discuss neutrino masses of the Dirac type. We consider the FCNC problem in Section 4. We find that the constraints on the mass scale is $m_{KK} \gtrsim 5 \times 10^5$ TeV for the Higgs field living in the bulk and $m_{KK} \gtrsim 700$ TeV for the Higgs field localized on the UV brane. This reiterates the power of the FCNC constraints in models without a built-in mechanism of the FCNC suppression.

2 Scenarios of electroweak symmetry breaking

We consider the Standard Model in the slice $y \in (0, \pi R)$ of 5D adS space-time with the metric (1). We will see in Section 3 that the fermion mass hierarchy problem is naturally solved provided that

$$e^{k\pi R} \gg 1 \ . \tag{2}$$

We treat $e^{k\pi R}$ as a large parameter in what follows.

The action for the Higgs field living in the 5D bulk is

$$S_5 = \int d^4x dy \sqrt{g} \Big(\frac{1}{2} g^{MN} \partial_M H \partial_N H - \frac{1}{2} m_H^2 H^2 - V(H) \Big) + S_b \ , \tag{3}$$

where m_H is the bulk Higgs mass, $V(H)$ is the symmetry breaking potential. The brane term S_b is added to have the zero mode in the absence of the potential $V(H)$ [5],

$$S_b = \Big(1 - \frac{\alpha}{2} \Big) k \int d^4x dy \sqrt{g} (\delta(y - \pi R) - \delta(y)) H^2 \ .$$

The constant α is tuned to $\alpha = \sqrt{4 + \frac{m_H^2}{k^2}}$, so that the zero mode exists. We will momentarily see that with the negative sign in front of α chosen in (4) and $\alpha > 1$, the Higgs zero mode is localized near the UV brane. This is the case we study in what follows.

Let us first switch off the potential $V(H)$, i.e., set $V(H) = 0$, and consider the free scalar field. One derives from the 5D action (3) with the brane term (4) the following equations of motion and boundary conditions,

$$\partial_\mu \partial^\mu H + e^{2ky} \partial_5 (e^{-4ky} \partial_5 H) + m_H^2 e^{-2ky} H = 0 \ , \tag{4}$$

$$\partial_5 H - (2-\alpha) k H|_{0,\ \pi R} = 0 \ . \tag{5}$$

Following the standard procedure, we expand the field H in the infinite sum:

$$H(x, y) = \sum_{n=0}^{\infty} h_n(x) H_n(y) \ , \tag{6}$$

where $h_n(x)$ are KK modes with masses m_n, while $H_n(y)$ are their bulk profiles. The zero mode is given by [5]

$$H_0(y) = N_0 e^{(2-\alpha)ky} \ . \tag{7}$$

It is clear from (3) that the effective profile is actually $e^{-ky} H_0(y)$. Hence, the zero mode is UV-localized for $\alpha > 1$. The normalization constant N_0 ensures the standard form of the kinetic term in the effective 4D action. The latter condition reads

$$\int_0^{\pi R} dy e^{-2ky} H_n^2(y) = 1 \ , \tag{8}$$

so that

$$N_0 = \sqrt{\frac{2k(\alpha - 1)}{1 - e^{2(1-\alpha)k\pi R}}} \approx \sqrt{2k(\alpha - 1)} \ . \tag{9}$$

The profiles of the excited KK modes are given by [5]

$$H_n(y) = N_n e^{2ky} \left[J_\alpha \left(\frac{m_n}{k} e^{ky} \right) + \frac{J_{\alpha-1}(\frac{m_n}{k})}{J_{-\alpha+1}(\frac{m_n}{k})} J_{-\alpha} \left(\frac{m_n}{k} e^{ky} \right) \right] \ , \tag{10}$$

with the normalization constants

$$N_n \approx \frac{m_n}{\sqrt{k}} \frac{1}{\sqrt{\int_0^{\beta_n} s J_\alpha^2(s) ds}} \ . \tag{11}$$

Here

$$\beta_n = \frac{m_n}{k} e^{k\pi R} \ . \tag{12}$$

The boundary conditions (5) determine the eigenvalues β_n and hence the masses of the KK excitations; these are found from

$$J_{\alpha-1}(\beta_n) = 0 \ . \tag{13}$$

Clearly, the lowest KK modes have $\beta_n \sim 1$ and hence

$$m_n \sim k e^{-k\pi R} \ . \tag{14}$$

Note that $\frac{m_n}{k}$ is a small parameter for not too large values of n in the regime (2) we consider.

To obtain the Higgs VEV, we turn on the potential $V(H)$. Let us begin with the choice

$$V(H) = -\frac{\mu^2}{2}H^2 + \lambda H^4 \ , \tag{15}$$

so that symmetry breaking occurs due to the bulk mass term. By inserting the KK decomposition (6) into the action (3) and integrating over the fifth coordinate, one obtains the effective 4D action. Assuming that the KK excitations are small, we treat the interaction between the zero modes and excited KK modes in the linear approximation in h_n and write

$$\begin{aligned} S_{eff} = \int d^4x \Big(& \frac{1}{2}(\partial h_0)^2 + \sum_{n=1}^{\infty} \frac{1}{2}(\partial h_n)^2 - \sum_{n=1}^{\infty} \frac{1}{2} m_n^2 h_n^2 + \frac{1}{2}\mu^2 (c_0 h_0^2 + \sum_{n=1}^{\infty} 2 c_n h_0 h_n) \\ & - \lambda (a_0 h_0^4 + \sum_{n=1}^{\infty} 4 a_n h_0^3 h_n) \Big) \ , \end{aligned} \tag{16}$$

where the constants a_0, a_n, c_0 and c_n are the overlap integrals

$$a_0 = \int_0^{\pi R} dy \sqrt{g} H_0^4 \ , \quad c_0 = \int_0^{\pi R} dy \sqrt{g} H_0^2 \ , \tag{17}$$

$$a_n = \int_0^{\pi R} dy \sqrt{g} H_0^3 H_n \ , \quad c_n = \int_0^{\pi R} dy \sqrt{g} H_0 H_n \ . \tag{18}$$

Making use of the effective action (16), we derive VEVs of the zero and excited KK modes:

$$v_0 = \sqrt{\frac{c_0 \mu^2}{4 a_0 \lambda}} \ , \tag{19}$$

$$v_n = \frac{c_n \mu^2 v_0 - 4 a_n \lambda v_0^3}{m_n^2} \ . \tag{20}$$

As the zero mode represents the standard Higgs field, we have: $v_0 = v_{SM} = 247$ GeV. We evaluate the integrals (17) and obtain

$$a_0 \approx \frac{N_0^4}{4(\alpha - 1)k} \ , \quad c_0 \approx \frac{N_0^2}{2\alpha k} \ . \tag{21}$$

We see that the constants a_0 and c_0 are estimated as $a_0 \sim k$ and $c_0 \sim 1$. Then the mass parameter μ responsible for symmetry breaking is of the order of the SM Higgs VEV, while $\lambda \lesssim k^{-1}$ in order that the effective 4D coupling $\lambda_4 = \lambda a_0$ be small.

Making use of (19) in (20), we write the KK VEVs as follows

$$v_n = (a_0 c_n - a_n c_0) \frac{v_0 \mu^2}{a_0 m_n^2} \ . \tag{22}$$

In what follows, we need the integrals (18) to the subleading order in $\frac{m_n}{k}$. The constants a_n are different at $\alpha > 2$ and $\alpha < 2$,

$$
\begin{aligned}
a_n &= -\frac{N_0^3 N_n}{2^\alpha \Gamma(\alpha) m_n}\left(\frac{m_n}{k}\right)^{\alpha-1}\left[1-\frac{(\alpha-1)}{2(\alpha-2)(2\alpha-3)}\left(\frac{m_n}{k}\right)^2\right] \qquad \alpha > 2\ , \\
a_n &= -\frac{N_0^3 N_n}{2^\alpha \Gamma(\alpha) m_n}\left(\frac{m_n}{k}\right)^{\alpha-1}\left[1-2^\alpha\Gamma(\alpha)\left(\frac{m_n}{k}\right)^{2(\alpha-1)}\int_0^{\beta_n} s^{3(1-\alpha)} J_\alpha(s) ds\right] \qquad \alpha < 2\ .
\end{aligned}
\tag{23}
$$

The constants c_n are given by

$$
c_n = -\frac{N_0 N_n}{\alpha 2^\alpha \Gamma(\alpha) m_n}\left(\frac{m_n}{k}\right)^{\alpha-1}\left[2(\alpha-1)+\left(\frac{m_n}{k}\right)^2 \ln\frac{m_n}{k}\right]\ . \tag{24}
$$

The constants a_n and c_n are estimated as $a_n \sim k\left(\frac{m_n}{k}\right)^{\alpha-1}$ and $c_n \sim \left(\frac{m_n}{k}\right)^{\alpha-1}$. Then a naive estimate of the KK VEVs would be $v_n \sim \left(\frac{v_{SM}}{m_n}\right)^2\left(\frac{m_n}{k}\right)^{\alpha-1} v_{SM}$. However, this is not the case. Indeed, to the leading order in $\frac{m_n}{k}$ the constants satisfy

$$
\frac{a_0}{c_0} = \frac{a_n}{c_n}\ . \tag{25}
$$

Taking into account the subleading terms in Eqs. (23), (24), we obtain the following expressions[1] for the KK VEVs at $\alpha > 2$ and $\alpha < 2$:

$$
v_n = \frac{1}{2^\alpha\Gamma(\alpha+1)}\sqrt{\frac{2(\alpha-1)}{\int_0^{\beta_n} sJ_\alpha^2(s)ds}}\left(\frac{\mu}{m_n}\right)^2\left(\frac{m_n}{k}\right)^{\alpha+1}\ln\frac{k}{m_n}v_0 \qquad \alpha > 2\ , \tag{26}
$$

$$
v_n = \sqrt{\frac{8(\alpha-1)^3}{\int_0^{\beta_n} sJ_\alpha^2(s)ds}}\left(\int_0^{\beta_n} s^{3(1-\alpha)}J_\alpha(s)ds\right)\left(\frac{m_n}{k}\right)^{3(\alpha-1)}\frac{\mu^2}{\alpha m_n^2}v_0 \qquad \alpha < 2\ . \tag{27}
$$

So, modulo factors of order one, the estimates are $v_n \sim \left(\frac{v_{SM}}{m_n}\right)^2\left(\frac{m_n}{k}\right)^{\alpha+1} v_{SM}$ in the case of the Higgs field localized with the parameter $\alpha > 2$ and $v_n \sim \left(\frac{v_{SM}}{m_n}\right)^2\left(\frac{m_n}{k}\right)^{3(\alpha-1)} v_{SM}$ for $\alpha < 2$.

So strong suppression is absent in a model with another mechanism of spontaneous symmetry breaking. Instead of the potential (3), one introduces

$$
V(H) = -\frac{1}{2}M\delta(y)H^2 + \lambda H^4\ , \tag{28}
$$

so that the mass term resides on the UV brane. Proceeding as before, we arrive at the following effective 4D action:

$$
\begin{aligned}
S_{eff} = \int d^4x\Big(&\frac{1}{2}(\partial h_0)^2 + \sum_{n=1}^{\infty}\frac{1}{2}(\partial h_n)^2 - \sum_{n=1}^{\infty}\frac{1}{2}m_n^2 h_n^2 + \frac{1}{2}M(h_0^2 H_0^2(0) + \\
&+ \sum_{n=1}^{\infty} 2h_0 h_n H_0(0) H_n(0)) - \lambda(a_0 h_0^4 + \sum_{n=1}^{\infty} 4a_n h_0^3 h_n)\Big)\ ,
\end{aligned}
\tag{29}
$$

[1] One can show that the terms omitted in (21) are negligible.

where a_0 and a_n are again the overlap integrals (17), (18). In this case, the VEVs are

$$v_0 = \sqrt{\frac{Mk}{4a_0\lambda}} \,, \tag{30}$$

$$v_n = \frac{MH_0(0)H_n(0)v_0 - 4a_n\lambda v_0^3}{m_n^2} \,. \tag{31}$$

We see from (30) that the mass M must be small, $M \sim \frac{v_{SM}^2}{k}$. Using this estimate as well as Eqs. (7)—(11), we find, modulo a factor of order one

$$v_n \sim \frac{v_{SM}^3}{m_n^2}\left(\frac{m_n}{k}\right)^{\alpha-1} \,. \tag{32}$$

It is straightforward to show that the cancellation between the leading order terms does not occur in Eq. (31), so the estimate (32) is indeed valid. However, this value is still very small from the viewpoint of physical applications discussed in Section 3.2.

3 Mass pattern of fermions

3.1 Quarks

The action for free bulk fermions is

$$S_5^\Psi = \int d^4x dy \sqrt{g}\left(ig^{MN}\bar{\Psi}\Gamma_M \nabla_N \Psi - m_\Psi \bar{\Psi}\Psi\right) \,, \tag{33}$$

where Γ_M are the 5D gamma matrices in adS space-time, ∇_M is the covariant derivative, m_Ψ is the fermion bulk mass. One chooses the fermions transforming as $\Psi(-y) = \pm\gamma_5\Psi(y)$ under the orbifold Z_2 symmetry, where the lower sign refers to $SU(2)_L$-doublets Q and the upper one to singlets u and d. As a result, there are no left zero modes of singlet quarks and right zero modes of doublets [5, 8] and one arrives at the SM chiral structure. Hereafter we consider zero modes of fermions. Their profiles are given by [5, 8]

$$Q_0(y) = N_L e^{(2-c_L)ky} \,, \quad u_0(y) = N_R^u e^{(2-c_R^u)ky} \,, \quad d_0(y) = N_R^d e^{(2-c^d)ky} \,. \tag{34}$$

The constants $c_{L,R}$ are related to the fermion bulk masses, $c_R = \frac{m_\Psi}{k}$, $c_L = -\frac{m_\Psi}{k}$, and the normalization constants are

$$N_L = \sqrt{\frac{(1-2c_L)k}{e^{(1-2c_L)k\pi R} - 1}} \,, \quad N_R^{u,d} = \sqrt{\frac{(1-2c_R^{u,d})k}{e^{(1-2c_R^{u,d})k\pi R} - 1}} \,. \tag{35}$$

It is worth noting that with the account of the warp factor in (33), the effective profiles of the zero modes are

$$\Psi_0 = Ne^{(1/2-c)ky} \,. \tag{36}$$

Hence, the zero modes are localized towards the IR and UV branes for $c < 1/2$ and $c > 1/2$, respectively. Now, assuming that the Higgs field lives in the bulk, we introduce its interaction with fermions,

$$S_5^q = \int d^4x dy \sqrt{g} \Big(\lambda_{ij}^d \bar{Q}_i H d_j + \lambda_{ij}^u \bar{Q}_i \widetilde{H} u_j + h.c. \Big) \ . \tag{37}$$

Neglecting the excited KK modes of the Higgs field for the time being and integrating Eq. (37) over the extra dimensional coordinate, we derive the effective 4D action:

$$S_{eff}^q = \int d^4x \Big(\lambda_{ij}^d I_{0ij}^d \bar{d}_{Li}(x) d_{Rj}(x) h_0(x) + \lambda_{ij}^u I_{0ij}^u \bar{u}_{Li}(x) u_{Rj}(x) h_0(x) + h.c. \Big) \ , \tag{38}$$

where $I_{0ij}^{u,d}$ are the overlap integrals of the zero modes of the Higgs and quark fields,

$$I_{0ij}^{u,d} = \int_0^{\pi R} dy \sqrt{g} H_0(y) Q_{0i}(y) d_{0j}(y) \ . \tag{39}$$

Explicitly,

$$I_{0ij}^{u,d} = N_{Li} N_{Rj}^{u,d} N_0 \frac{1 - e^{(2-\alpha-c_{Li}-c_{Rj}^{u,d})k\pi R}}{(\alpha + c_{Li} + c_{Rj}^{u,d} - 2)k} \ . \tag{40}$$

In the low-energy theory, the action (37) leads to the quark mass matrix,

$$M_{ij}^{u,d} = \lambda_{ij}^{u,d} I_{0ij}^{u,d} v_{SM} \ . \tag{41}$$

In our calculations we assume that the condition $2 - \alpha - c_{Li} - c_{Rj} < 0$ is satisfied, so that the exponential factor in Eq. (40) can be neglected. Furthermore, we do not introduce the hierarchy between the 5D Yukawa couplings and set $\lambda_{i,j}^{u,d} \sim k^{-1/2}$. Then, using Eq. (41), we estimate the elements of the mass matrix as follows,

$$M_{ij}^{u,d} \sim N_{Li} N_{Rj}^{u,d} \frac{v_{SM}}{k} \ . \tag{42}$$

The hierarchy between quark masses and mixings is generated by the hierarchy between the normalization constants N_{Li}, N_{Rj}, which in turn is due to zero mode profiles. Aiming at diagonalizing the mass term, we perform the unitary transformations of left and right quarks (up- and down-quarks independently) with corresponding matrices A_L and A_R. The latter satisfy the conditions

$$A_L M M^\dagger A_L^{-1} = M_{diag}^2 \ , \quad A_R M^\dagger M A_R^{-1} = M_{diag}^2 \ . \tag{43}$$

These ensure that the quark mass matrix

$$m = A_L M A_R^{-1} \tag{44}$$

is diagonal.

With no hierarchy between the Yukawa couplings, the Hermitean matrix $MM^\dagger$ has the following structure,

$$(MM^\dagger)_{ij} \sim N_{Li} N_{Lj} \sum_{k=1}^{3} N_{Rk}^2 \frac{v_{SM}^2}{k^2} \ . \tag{45}$$

So, only the normalization constants of the doublets N_{Li} are responsible for the hierarchy in Eq. (45). We order them as follows: $N_{L1} \ll N_{L2} \ll N_{L3}$. Then the elements of the matrix A_L are estimated by the the ratios of the constants N_{Li}:

$$A_L \sim \begin{pmatrix} 1 & \frac{N_{L1}}{N_{L2}} & \frac{N_{L1}}{N_{L3}} \\ \frac{N_{L1}}{N_{L2}} & 1 & \frac{N_{L2}}{N_{L3}} \\ \frac{N_{L1}}{N_{L3}} & \frac{N_{L2}}{N_{L3}} & 1 \end{pmatrix} \ . \tag{46}$$

By analogy, the Hermitean matrix $M^\dagger M$ is estimated as

$$M^\dagger M \sim N_{Ri} N_{Rj} \sum_{k=1}^{3} N_{Lk}^2 \frac{v_{SM}^2}{k^2} \ . \tag{47}$$

Then, assuming the hierarchy of the normalization constants, $N_{R1} \ll N_{R2} \ll N_{R3}$, we obtain

$$A_R \sim \begin{pmatrix} 1 & \frac{N_{R1}}{N_{R2}} & \frac{N_{R1}}{N_{R3}} \\ \frac{N_{R1}}{N_{R2}} & 1 & \frac{N_{R2}}{N_{R3}} \\ \frac{N_{R1}}{N_{R3}} & \frac{N_{R2}}{N_{R3}} & 1 \end{pmatrix} \ . \tag{48}$$

Formally, this estimate is valid also for $N_{R1} \sim N_{R2} \sim N_{R3}$, as is the case for down-quarks (see below). Finally, using the estimates (42), (46) and (48), we estimate the quark masses:

$$m_i^{u,d} \sim N_{Li} N_{Ri}^{u,d} \frac{v_{SM}}{k} \ . \tag{49}$$

Now, let us consider flavor mixing in the quark sector, which is described by the Cabibbo–Kobayashi–Maskawa matrix. The latter is given by

$$C = A_L^d (A_L^u)^{-1} \ .$$

Since the matrices A_L^d and A_L^u have one and the same general structure given by Eq. (46), the CKM matrix is also estimated by the right hand side of Eq. (46). We now recall the entries of the CKM matrix,

$$|C| = \begin{pmatrix} 0.97 & 0.23 & 0.0040 \\ 0.23 & 0.97 & 0.042 \\ 0.0081 & 0.041 & 0.99 \end{pmatrix} \ , \tag{50}$$

and compare them with Eq. (46). We see that the right pattern is obtained for

$$\frac{N_{L1}}{N_{L2}} \approx \frac{1}{10} \ , \quad \frac{N_{L2}}{N_{L3}} \approx \frac{1}{25} \ . \tag{51}$$

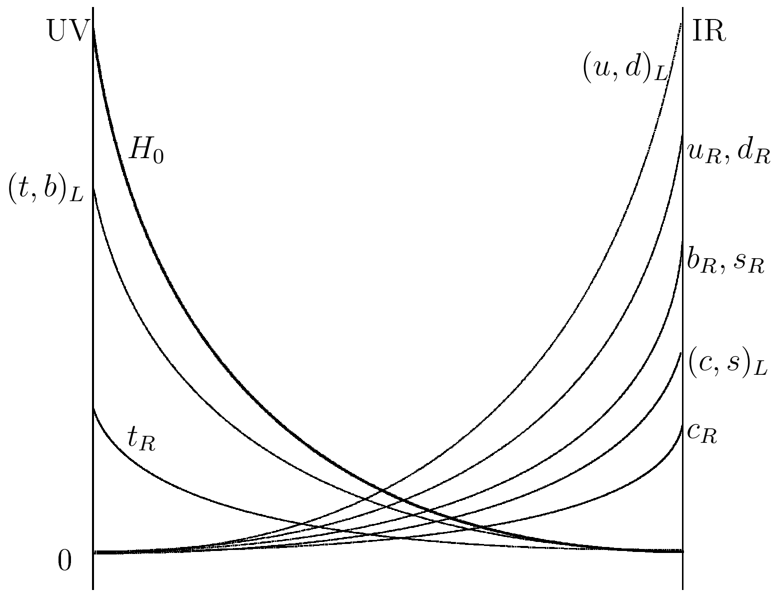

Figure 1: Effective profiles (see Eq. (36)) of quark zero modes.

The estimate (49) is consistent with known experimental values of the quark masses [21] $m_u \approx 2.6$ MeV, $m_d \approx 5.0$ MeV, $m_c \approx 1.3$ GeV, $m_s \approx 100$ MeV, $m_b \approx 4.2$ GeV, $m_t \approx 171$ GeV, provided that

$$\frac{N^u_{R1}}{N^u_{R2}} \approx \frac{1}{30}\,, \quad \frac{N^u_{R2}}{N^u_{R3}} \approx \frac{1}{5}\,, \quad \frac{N^d_{R1}}{N^d_{R2}} \approx \frac{1}{2}\,, \quad \frac{N^d_{R2}}{N^d_{R3}} \approx 1\,, \quad \frac{N^d_{R3}}{N^u_{R3}} \approx \frac{1}{40}\,. \tag{52}$$

The overall scale of the normalization constants is obtained by requiring that the mass of the top quark has the correct value. This gives $N_{L3}N^u_{R3} \simeq k$. As an example, we choose

$$N^u_{L3} = 1.3\sqrt{k}\,, \quad N^u_{R3} = 0.6\sqrt{k}\,. \tag{53}$$

Finally, using formulas (35), one estimates the dimensionless bulk masses of quarks, which we denoted by c. Their values for the warp factor $k\pi R = 10$ are given in Table 1. We see explicitly that $c_{L,R} < 1/2$ for the lightest four quarks. As we noticed above, this means that they live near the IR brane, as expected. Qualitative picture of the quark localization is shown in Fig. 1.

We have still to choose the set of 5D Yukawa couplings $\lambda^{u,d}_{ij}\sqrt{k}$. Allowing them to vary within the interval (1/3, 3), one can adjust masses and flavor mixings. Let us set

$$\lambda^u_{ij}\sqrt{k} = \begin{pmatrix} 1.2 & 0.4 & -1.9 \\ 1.7 & 1.1 & -0.9 \\ -0.8 & 0.6 & 1.3 \end{pmatrix}\,, \tag{54}$$

$$\lambda^d_{ij}\sqrt{k} = \begin{pmatrix} 0.9 & -0.4 & 1.3 \\ -1.3 & 1.3 & -0.5 \\ 1.8 & 0.3 & 1.1 \end{pmatrix}\,, \tag{55}$$

where we ignore phases for the sake of simplicity. With these 5D Yukawa couplings one obtains the mixing matrix

$$|C| = \begin{pmatrix} 0.97 & 0.25 & 0.0040 \\ 0.25 & 0.97 & 0.040 \\ 0.0010 & 0.040 & 0.998 \end{pmatrix} \tag{56}$$

and quark masses $m_u \approx 3$ MeV, $m_d \approx 9$ MeV, $m_c \approx 1.5$ GeV, $m_s \approx 90$ MeV, $m_b \approx 4.5$ GeV, $m_t \approx 170$ GeV. Obviously, all these values are in a reasonable agreement with the experimental data.

c	Q	u	d
c_{L1}	-0.1	-	-
c_{L2}	0.2	-	-
c_{L3}	1.3	-	-
c_{R1}	-	0.0	0.0
c_{R2}	-	0.4	0.1
c_{R3}	-	0.7	0.1

Table 1: Quark parameters in the 5D SM with warp factor $k\pi R = 10$.

3.2 Leptons

The interaction of leptons with the Higgs field in the 5D bulk is

$$S_5^l = \int d^4x dy \sqrt{g} \Big(\lambda^l_{ij} \bar{L}_i l_j + \lambda^\nu_{ij} \bar{L}_i \widetilde{H} \nu_j + h.c. \Big) \ , \tag{57}$$

where $L_i(x,y)$ are the lepton $SU(2)_L$-doublets, $l_j(x,y)$ and $\nu_j(x,y)$ are singlet charged and neutral leptons, respectively, and λ^l_{ij} and λ^ν_{ij} are their 5D Yukawa couplings.

This interaction generates the neutrino masses of the Dirac type. An interesting possibility here would be that neutrinos obtain their masses predominantly via the interaction with excited KK modes of the Higgs field. Then the smallness of the neutrino masses would be due to the suppression of VEVs of the Higgs KK excitations. Let us see, however, that this mechanism does not work in the model we discuss. To this end, we keep all modes in the decomposition of the Higgs field (6). Inserting the latter into Eq. (57), we derive the neutrino mass matrix in the low-energy limit,

$$M^\nu_{ij} = \lambda^\nu_{ij} \Big(v_0 I^\nu_{0ij} + \sum_{n=1}^{\infty} v_n I^\nu_{nij} \Big) \ , \tag{58}$$

where v_0 and v_n are VEVs of the zero and excited modes of the Higgs field, as described in Section 2; I^ν_{0ij} and I^ν_{nij} are the overlap integrals of appropriate wave functions. The first integral is given by the Eq. (40) with the substitution $u \to \nu$, while the second one is

$$I^\nu_{nij} = \int_0^{\pi R} dy L_{io}(y) H_n(y) \nu_j(y) \ , \tag{59}$$

where $H_n(y)$ is given by Eq. (10). Explicitly,

$$I^{\nu}_{nij} = N_n N_{Li} N^{\nu}_{Rj} \frac{1}{m_n} \left(\frac{m_n}{k}\right)^{c_{Li}+c^{\nu}_{Rj}-1} \int_{\frac{m_n}{k}}^{\beta_n} s^{1-c_{Li}-c^{\nu}_{Rj}} J_{\alpha}(s) ds \; . \tag{60}$$

Hereafter we assume that $c_L + c^{\nu}_R < 2 + \alpha$, so that the integral here is of order 1. One can show that our basic conclusion is valid in the opposite case as well.

The expression (58) shows that neutrinos obtain their masses due to their interactions with both zero mode and excited modes of the Higgs field. The excited mode contribution would dominate for

$$v_0 I^{\nu}_0 \ll \sum_{n=1}^{\infty} v_n I^{\nu}_n \; . \tag{61}$$

Omitting summation and using (40) and (60), we rewrite Eq. (61) as follows,

$$|e^{(2-\alpha-c_L-c^{\nu}_R)k\pi R} - 1| \ll e^{(1-c_L-c^{\nu}_R)k\pi R} \frac{v_n}{v_0} \; . \tag{62}$$

Here we use the fact that $\frac{m_n}{k} e^{k\pi R} \sim 1$ for the lightest KK modes. The condition (62) is equivalent to the following two,

$$e^{(c_L+c^{\nu}_R-1)k\pi R} \ll \frac{v_n}{v_0} \tag{63}$$

and

$$e^{(1-\alpha)k\pi R} \ll \frac{v_n}{v_0} \; , \tag{64}$$

which must be satisfied simultaneously. We now recall the expressions (26) and (31) for VEVs of the excited Higgs modes, and find that in both scenarios of spontaneous symmetry breaking considered in Section 2, the inequality (64) is not satisfied. In the best case, the contribution of the Higgs KK excitations is suppressed by the small factor $\frac{v^2_{SM}}{m^2_n}$ as compared to the zero mode.

Thus, all lepton masses are obtained via the interaction with the zero mode of the Higgs field. Still, the picture here is rather different as compared to the quark sector. Indeed, the lepton mixing matrix does not exhibit strong hierarchy [21],

$$|C| \approx \begin{pmatrix} 0.79-0.88 & 0.47-0.61 & <0.18 \\ 0.19-0.52 & 0.42-0.73 & 0.52-0.82 \\ 0.20-0.53 & 0.44-0.74 & 0.56-0.81 \end{pmatrix} \; .$$

Similarly to the case of quarks, we estimate it by the right hand side of Eq. (46). Then we conclude that the normalization constants N_L are of one and the same order. It is therefore natural to assume that all $N_L \sim \sqrt{k}$, so that the dimensionless bulk masses of lepton doublets $c_L > 1/2$, i.e., the doublets reside near the UV brane. Otherwise, we would need to fine tune the parameters c_L to be very close to each other.

Obviously, up to the change of notations $u \to \nu$ and $d \to l$, the estimate Eq. (49) remains valid for leptons. By choosing the normalization constants of singlet fermions N_R and thus

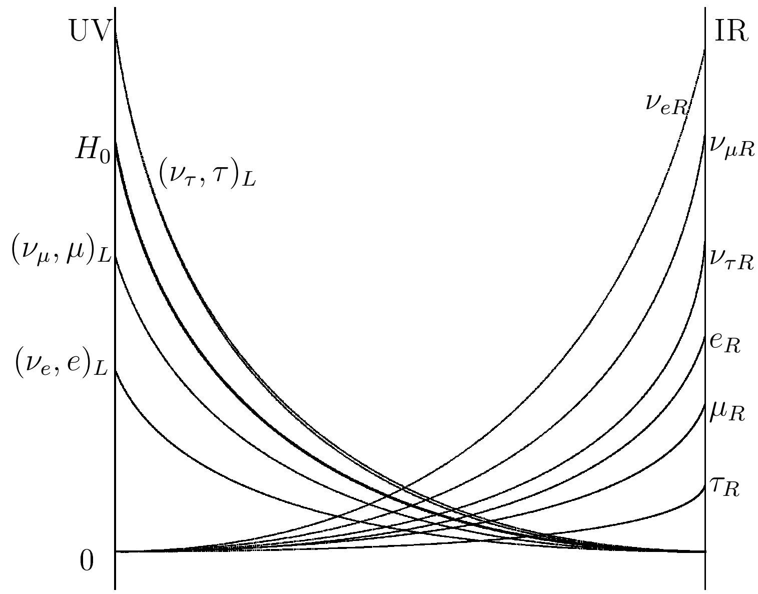

Figure 2: Effective profiles (see Eq. (36)) of lepton zero modes.

the parameters c_R^l and c_R^ν in an appropriate way, one adjusts 4D masses of leptons. Since we assume that $N_L \sim \sqrt{k}$, the constants N_R must be small, $N_R \ll \sqrt{k}$, in order that the lepton masses be small compared to the Higgs VEV. Hence, all c_R^l and c_R^ν must be smaller than 1/2. So, we come to the assignment that all singlet leptons reside towards the IR brane. Finally, in Table 2 we present the set of the 5D parameters c leading to the correct hierarchy of lepton masses. We choose the normal hierarchy of neutrino masses and assume no degeneracy, i.e. $m_1 \ll m_2$, $m_2 \approx \sqrt{\Delta m^2_{sol}} \approx 0.008$ eV and $m_3 \approx \sqrt{\Delta m^2_{atm}} \approx 0.05$ eV. The qualitative picture of lepton localization in the 5D bulk is shown in Fig. 2.

c	L	ν	e
c_{L1}	1.0	-	-
c_{L2}	2.0	-	-
c_{L3}	3.0	-	-
c_{R1}	-	< -2.1	-0.8
c_{R2}	-	-2.1	-0.3
c_{R3}	-	-1.8	0.1

Table 2: Lepton parameters in the 5D SM with warp factor $k\pi R = 10$.

To conclude, masses and mixings in both quark and lepton sectors are reproduced in the model we discuss without introducing large or small parameters. The profiles of the fermion and Higgs zero modes are naturally steep in the warped fifth dimension, which translates into the strong hierarchies of masses and quark mixings in the 4D world. The non-hierarchical pattern of neutrino mixings is also natural with our choice of the localization of the left lepton doublets.

4 Kaon mixing

4.1 Kaon mixing mediated by excited Higgs field

Unlike in the case of the Higgs field localized towards the IR brane, the FCNC suppression is not at all automatic in models we consider. The main source of FCNC in the model with the bulk Higgs field, whose zero mode is localized near the UV brane, is the interaction of light quarks with the KK excitations of the Higgs field. This interaction is fairly strong because both zero modes of light quarks and the Higgs KK modes are large near the IR brane, so they overlap substantially.

Let us consider in detail the interaction of light down-quarks with the KK excitations of the Higgs field. Since this interaction is not diagonal in the flavor space, it leads to kaon mixing, which is severely constrained by experiment. Integrating the relevant terms in the action (37) over the fifth coordinate, we arrive at the effective 4D action,

$$S^{eff}_{quark} = \int d^4x \Big(\sum_{n=1}^{\infty} y^d_{nij} \bar{d}_{Li}(x) d_{Rj}(x) h_n(x) + h.c. \Big) \ . \tag{65}$$

Here $y^{u,d}_{nij}$ are the effective 4D Yukawa couplings. They are given by

$$y_{nij} = \lambda^d_{ij} I^d_{nij} \ , \tag{66}$$

where I^d_{nij} are the overlap integrals of the Higgs KK excitations and zero modes of the singlet down-quarks and quark doublets,

$$I^d_{nij} = \int dy \sqrt{g} Q_{0i}(y) H_n(y) d_{0j}(y) \ . \tag{67}$$

Explicitly,

$$I^d_{nij} = N_{Li} N^{u,d}_{Rj} N_n \frac{1}{m_n} \left(\frac{m_n}{k}\right)^{c_{Li}+c^{u,d}_{Rj}-1} \int_0^{\beta_n} s^{1-c_{Li}-c^{u,d}_{Rj}} J_\alpha(s) ds \ . \tag{68}$$

As shown in Section 3.1, the lightest down-quarks reside towards the IR brane. Thus, their normalization constants are

$$N_{Li} \approx \sqrt{(1-2c_{Li})k} e^{(c_{Li}-1/2)k\pi R}, \quad N^d_{Rj} \approx \sqrt{(1-2c^d_{Rj})k} e^{(c^d_{Rj}-1/2)k\pi R} \ . \tag{69}$$

In this way we obtain the 4D Yukawa couplings of s- and d-quarks,

$$y_{nij} \approx \lambda^d_{ij} \sqrt{\frac{(1-2c_{Li})(1-2c^d_{Rj})k}{\int_0^{\beta_n} s J^2_\alpha(s) ds}} \int_0^{\beta_n} \left(\frac{s}{\beta_n}\right)^{1-c_{Li}-c^d_{Rj}} J_\alpha(s) ds \ . \tag{70}$$

Here the flavor indices are $i, j = 1, 2$, and the integrals are of order 1. Hence, the Yukawa couplings are unsuppressed, $y_{nij} \sim 1$. To obtain the Yukawa couplings of the physical quark

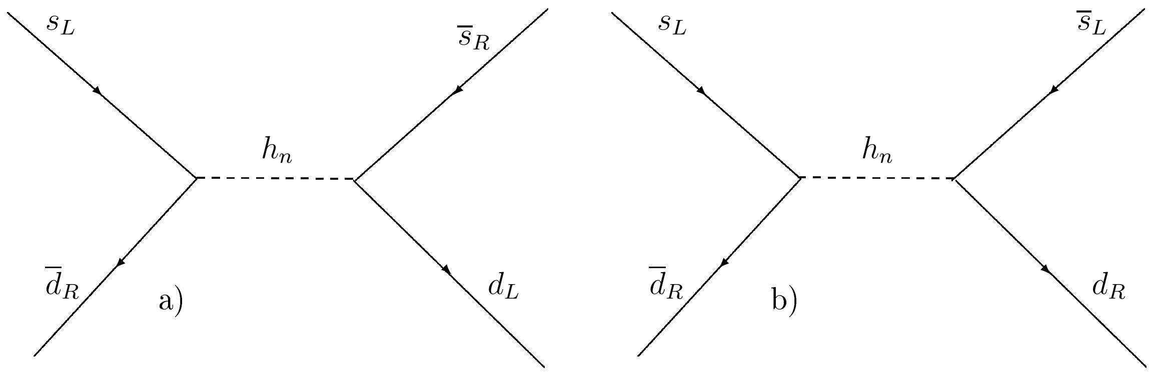

Figure 3: Excited Higgs mediated processes leading to kaon mixing.

states, we perform the rotation of the quark fields with the matrices A_L and A_R. We obtain in the physical basis

$$\begin{aligned} y'_{n12} &\approx a_{L11}(y_{n11}a^{-1}_{R12} + y_{n12}a^{-1}_{R22}) \ , \\ y'_{21} &\approx a_{L22}(y_{n21}a^{-1}_{R11} + y_{n22}a^{-1}_{R21}) \ , \end{aligned} \tag{71}$$

where we neglect the b-quark contribution; the constants a are the entries of the matrices A_L and A_R estimated by Eqs. (46), (48). Obviously, the physical Yukawa couplings are also unsuppressed, $y'_{n12} \sim y'_{n21} \sim 1$. This precisely means that the RS-GIM mechanism does not work in the case of the Higgs field residing near the UV brane. Consequently, the only way to suppress dangerous FCNC is to assume that the Higgs KK excitations have very large masses.

Generally, $\Delta F = 2$ processes are described by the following Hamiltonian:

$$H^{\Delta F=2}_{eff} = \sum_{a=1}^{5} C_a Q_a^{q_i q_j} + \sum_{a=1}^{3} \widetilde{C}_a \widetilde{Q}_a^{q_i q_j} \ . \tag{72}$$

The four-fermion operators Q_a are given by

$$\begin{gathered} Q_1^{q_i q_j} = \bar{q}^{\alpha}_{jL}\gamma_\mu q^{\alpha}_{iL}\bar{q}^{\beta}_{jL}\gamma^\mu q^{\beta}_{iL}, \quad Q_2^{q_i q_j} = \bar{q}^{\alpha}_{jR} q^{\alpha}_{iL}\bar{q}^{\beta}_{jR} q^{\beta}_{iL}, \quad Q_3^{q_i q_j} = \bar{q}^{\alpha}_{jR} q^{\beta}_{iL}\bar{q}^{\beta}_{jL} q^{\alpha}_{iR} \ , \\ Q_4^{q_i q_j} = \bar{q}^{\alpha}_{jR} q^{\alpha}_{iL}\bar{q}^{\beta}_{jL} q^{\beta}_{iR}, \quad Q_5^{q_i q_j} = \bar{q}^{\alpha}_{jR} q^{\beta}_{iL}\bar{q}^{\beta}_{jL} q^{\alpha}_{iR} \ . \end{gathered} \tag{73}$$

The operators $\widetilde{Q}_a$ are obtained by the interchange $L \leftrightarrow R$. Choosing $q_j, q_i = d, s$, we focus on kaon mixing. The exchange by the KK excitations of the Higgs field contributes to the coefficients C_2, $\widetilde{C}_2$ and C_4,

$$C_2 = \sum_{n=1}^{\infty} \frac{({y'}^{\star}_{n21})^2}{m_n^2} \ , \quad \widetilde{C}_2 = \sum_{n=1}^{\infty} \frac{(y'_{n12})^2}{m_n^2} \ , \quad C_4 = \sum_{n=1}^{\infty} \frac{2{y'}_{n12}{y'}^{\star}_{n21}}{m_n^2} \ . \tag{74}$$

This is shown in Fig. 3, where Figs. 3a and 3b correspond to C_2 and C_4, respectively. The imaginary parts of the coefficients C are responsible for CP-violating mixing of kaons K_1^0 and K_2^0. Within our model, there is no natural way to suppress the phases of the coefficients C. Thus, we assume that Im$C \sim C$.

The experimental constraints on the imaginary parts are [22]

$$-5.1 \times 10^{-17}\ \mathrm{GeV}^{-2} \lesssim \ \mathrm{Im}C_2,\ \mathrm{Im}\widetilde{C}_2 \lesssim 9.3 \times 10^{-17}\ \mathrm{GeV}^{-2}\ , \tag{75}$$

$$-1.8 \times 10^{-17}\ \mathrm{GeV}^{-2} \lesssim \ \mathrm{Im}C_4 \lesssim 0.9 \times 10^{-17}\ \mathrm{GeV}^{-2}\ . \tag{76}$$

By comparing Eq. (75) with Eq. (74) and recalling that $y'_{nij} \sim 1$, we see that the masses of the Higgs KK excitations must be very large:

$$m_n \gtrsim 5 \times 10^5\ \mathrm{TeV}\ . \tag{77}$$

The real parts of the coefficients C contribute to the kaon mass difference. The corresponding constraints are three orders of magnitude weaker than (75) and (76). Using these constraints, we find that irrespectively of the above assumption Im$C \sim C$, the masses of the Higgs KK excitations must obey $m_n \gtrsim 10^4$ TeV.

4.2 Other sources of kaon mixing

A possible way to avoid the constraint (77) is to assume that the Higgs field is localized on the UV brane. Hence, it does not have KK excitations at all, and the analysis of Section 4.1 does not apply. In this case, the major source of kaon mixing is the interaction of down-quarks with the KK excitations of the bulk gauge fields. For simplicity, let us consider the interaction with the bulk photons; exchange by the KK Z-bosons is treated in a similar way and yields analogous results. The relevant part of the 5D action is given by

$$S_5^\gamma = e_5 \int d^4x dy \sqrt{g} \Big(\bar{Q}_i g^{MN} \Gamma_M A_N Q_i + \bar{d}_i g^{MN} \Gamma_M A_N d_i \Big)\ . \tag{78}$$

As usual, we expand the gauge field in the tower of KK modes,

$$A^\mu(x,y) = \sum_{n=0}^{\infty} a_n^\mu(x) A_n(y)\ . \tag{79}$$

The zero mode of the bulk electromagnetic field is flat [23, 24],

$$A_0(y) = \frac{1}{\sqrt{\pi R}}\ . \tag{80}$$

Therefore, the 4D electric charge is $e = \frac{e_5}{\sqrt{\pi R}}$. The profiles of the KK excitations are [23, 24]

$$A_n(y) = N_n^\gamma e^{ky} \left[J_1\Big(\frac{m_n^\gamma}{k} e^{ky}\Big) + C_n Y_1\Big(\frac{m_n^\gamma}{k} e^{ky}\Big) \right]\ , \tag{81}$$

where m_n^γ denote the masses of the KK excitations. The normalization factor N_n^γ is given by

$$N_n^\gamma \approx \frac{m_n^\gamma}{\sqrt{k}} \frac{1}{\sqrt{\int_0^{\gamma_n} s J_1^2(s) ds}} , \tag{82}$$

while the constant C_n is

$$C_n = - \frac{J_1\left(\frac{m_n^\gamma}{k}\right) + \frac{m_n^\gamma}{k} J_1'\left(\frac{m_n^\gamma}{k}\right)}{Y_1\left(\frac{m_n^\gamma}{k}\right) + \frac{m_n^\gamma}{k} Y_1'\left(\frac{m_n^\gamma}{k}\right)} . \tag{83}$$

The masses of the KK excitations of the electromagnetic field are determined from the following eigenvalue equation [23, 24]

$$\frac{m_n^\gamma e^{k\pi R}}{k} = \gamma_n, \quad J_1\left(\frac{m_n^\gamma}{k} e^{k\pi R}\right) = 0 . \tag{84}$$

Hereafter we omit the zero mode in the decomposition (79), since the interaction with the zero mode is universal in the flavor space and does not give rise to flavor violating processes. By inserting Eq. (79) into the 5D action (78) and integrating the latter over the fifth coordinate, we arrive at the following effective 4D action:

$$S_{eff}^\gamma = \int d^4x \Big(\sum_{n=1}^{\infty} b_{nij}^L \bar{d}_{Li}(x) \gamma_\mu d_{Lj}(x) a_n^\mu(x) + (L \leftrightarrow R) \Big) . \tag{85}$$

The constants $b_{nij}^{L(R)}$ are the effective 4D couplings of left (right) down-quarks with the n-th KK excitation. These couplings are obtained from the initial 5D theory,

$$b_{nij}^{L(R)} = e_5 W_{nij}^{L(R)} , \tag{86}$$

where the constants $W_{nij}^{L(R)}$ are the overlap integrals of the appropriate wave functions,

$$W_{nij}^L = \delta_{ij} \int dy \sqrt{g} e^{ky} A_n(y) Q_{Li}(y) Q_{Lj}(y) \tag{87}$$

and the analogous expression for the constants W_{nij}^R. By performing the integration in Eq. (87) and using Eq. (86), we obtain

$$b_{nij}^L = e_5 \delta_{ij} N_{Li} N_{Lj} N_n \frac{1}{m_n^\gamma} \left(\frac{m_n^\gamma}{k}\right)^{c_{Li}+c_{Lj}-1} \int_0^{\gamma_n} s^{1-c_{Li}-c_{Lj}} J_1(s) ds . \tag{88}$$

The expression for the couplings b_{nij}^R is obtained by the replacing $N_{Li} \to N_{Ri}^d$ and $c_{Li} \to c_{Rj}^d$. As in the previous case, all RS suppression factors disappear for $i, j = 1, 2$. Thus, the couplings $b_{n11}^{L(R)}$ and $b_{n22}^{L(R)}$ are of the order of the 4D electromagnetic coupling e.

The effective action (85) is diagonal in the quark fields. The flavor violating terms appear upon the rotation of the quark fields by the matrices A_L and A_R,

$$S^{eff}_{K-\widetilde{K}} = \int d^4x \Big(\sum_{n=1}^{\infty} {b'}^L_{n12} \bar{d}_L(x)\gamma_\mu s_L(x) a^\mu_n(x) + \sum_{n=1}^{\infty} {b'}^L_{n21} \bar{s}_L(x)\gamma_\mu d_L(x) a^\mu_n(x) + (L \leftrightarrow R) \Big) \ , \quad (89)$$

where, according to Eqs. (46), (48), the couplings ${b'}^{L(R)}_{n12}$ and ${b'}^{L(R)}_{n21}$ are estimated as

$$|{b'}^L_{n12}| \sim |{b'}^L_{n21}| \sim |b^L_{n11} - b^L_{n22}| \frac{N_{L1}}{N_{L2}} \quad (90)$$

and

$$|{b'}^R_{n12}| \sim |{b'}^R_{n21}| \sim |b^R_{n11} - b^R_{n22}| \frac{N^d_{R1}}{N^d_{R2}} \ . \quad (91)$$

Note that the constants ${b'}^L_{n12}$ and ${b'}^L_{n21}$ are suppressed as compared to the initial ones b^L_{n11} and b^L_{n22}, which is a consequence of the smallness of the factor $\frac{N_{L1}}{N_{L2}}$. Note also that the profiles of d_R and s_R can be chosen very similar to each other, so that $b^R_{n11} \approx b^R_{n22}$ and hence b'_{n12} and b'_{n21} can be made very small. For the warp factor $k\pi R = 10$ and the parameters c listed in Table 1, we obtain $|{b'}^L_{n21}| \sim |{b'}^L_{n12}| \sim \frac{1}{50}$ and $|{b'}^R_{n12}| \sim |{b'}^R_{n21}| \sim \frac{1}{30}$.

The exchange by the KK excitations of the electromagnetic field gives contribution to the coefficients C_1 and $\widetilde{C}_1$,

$$C_1 = \sum_{n=1}^{\infty} \frac{({b'}^L_{n12})^2}{m^{\gamma 2}_n} \ , \quad \widetilde{C}_1 = \sum_{n=1}^{\infty} \frac{({b'}^R_{n12})^2}{m^{\gamma 2}_n} \ . \quad (92)$$

Their imaginary parts are constrained as follows [22],

$$-4.4 \times 10^{-15}\ \mathrm{GeV}^{-2} \lesssim \ \mathrm{Im}C_1,\ \mathrm{Im}\widetilde{C}_1 \lesssim 2.8 \times 10^{-15}\ \mathrm{GeV}^{-2} \ . \quad (93)$$

These bounds imply the following constraint on the masses of the photon KK excitations:

$$m^\gamma_n \gtrsim 700\ \mathrm{TeV} \ . \quad (94)$$

We see that the masses of the KK excitations can be three orders of magnitude smaller than the ones in the bulk Higgs scenario. However, their values are still out of reach of future experiments.

Besides the exchange by the KK modes, there are other sources of FCNC. They are important in the case of the IR-localized Higgs field, but subdominant in our scenario. One of these sources is the interaction of down-quarks with the zero mode of the Z-boson. The profile of the latter is not exactly flat [23, 24]. However, the deviation from the flatness is of the order of the ratio $\frac{m_Z^2}{m_{KK}^2}$, where m_{KK} is the typical mass scale of the KK excitations. Accordingly, flavor-violating vertices are suppressed by $\frac{m_Z^2}{m_{KK}^2}$. With m_{KK} constrained by Eqs. (77) or (94), the contribution to the coefficient C_1 coming from the interaction with the zero mode of the Z-boson is negligibly small, $C_1 \sim \frac{m_Z^2}{M_{KK}^4}$.

As described in [25], there are also flavor violating processes mediated by the zero mode of the Higgs field. They occur due to the interaction with the KK excitations of fermions. These processes give negligibly small contribution to the coefficients C for the same reason as above.

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